A CONTRIBUTION TO THE THEORETICAL ANALYSIS OF PERFORATED SHEAR WEBS

BY

CHU-KIA WANG

B. S., St. John's University, 1938

M. S., University of Colorado, 1942

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN ENGINEERING
IN THE GRADUATE SCHOOL OF THE
UNIVERSITY OF ILLINOIS, 1945

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I. INTRODUCTION

A flat, finite, rectangular plate under the action of pure uniform shearing stress along its four edges is a common structural member. Such a plate may be called a shear web. For purposes of weight saving or access, one or more circular holes may be made in the middle of the shear web. The holes may be plain or reinforced around the rim in various ways. Shear webs with plain or reinforced circular holes in them are called perforated shear webs.

The determination of the stress components at any point of a perforated shear web presents itself as a problem of theoretical analysis. The present investigation deals with the stress analysis of a finite rectangular shear panel with a circular hole, either plain or reinforced in various ways, in the center.

II. THEORY

The stress function F for a rectangular shear web with a circular hole, which is either plain or reinforced with rim only, can be expressed as the sum of an infinite series in $\sin 2n\Theta$ with four arbitrary coefficients in each term. If a finite number of terms, i, is considered, then 2i of the conditions required for the determination of the 4i arbitrary coefficients are supplied by the boundary conditions at the rim of the hole. The assumption that the area integral of the square of the difference between the true $\nabla^2 F$ and that as approximated by the sine series shall be a minimum, furnishes the other 2i necessary conditions. An application of Green's Theorem in the Plane reduces this area integral to a line integral around the boundary. Consequently the boundary conditions at the edges are only approximately satisfied. The compatibility condition, by the nature of the series expression of the stress function, is always satisfied.

When the hole is reinforced with ring only or with both rim and ring, two different stress functions will have to be assumed: one for the ring and the other for the region outside of ring. Therefore there are 8i arbitrary coefficients to be determined: 4i from the conditions discussed in the preceding paragraph and 4i from the conditions of continuity of stresses and displacements at the outer circumference of the ring.

III. NUMERICAL APPLICATIONS

Numerical computation is done only for a square shear panel, which is anti-symmetrical with respect to the x- and y-axes and symmetrical with respect to the lines $\Theta = \pm \pi/4$. Only two terms of the series, n=1 and n=3, are used in all examples. The variables considered in the different cases are:

(a) Square shear panel with a plain circular hole- 5 examples; Hole diameter = 0.1, 0.2, 0.3, 0.4, or 0.5 of panel length.

(b) Square shear panel with a circular hole reinforced with rim only- 15 examples; Hole diameter = 0.1, 0.2, 0.3, 0.4, or 0.5 of panel length; Amount of reinforcing = 40%, 100%, or 200% of original hole material.

(c) Square shear panel with a circular hole reinforced with ring only- 6 examples; Hole diameter = 0.5 of panel length; Amount of reinforcing = 40%, 100%, or 200% of original hole material; Thickness of ring = 3 or 2 times that of plate.

(d) Square shear panel with a circular hole reinforced with both rim and ring- 6 examples; Hole diameter = 0.5 of panel length; Amount of reinforcing (equally divided between rim and ring) = 40%, 100%, or 200% of original hole material; Thickness of ring = 3 or 2 times that of plate.

IV. CHECK ON BOUNDARY CONDITIONS

It can be shown that the fundamental assumption, that

the area integral of the square of error in $\nabla^2 F$ shall be a minimum, must have something to do with the external boundary, and that this area integral can be reduced to a line integral around the boundary by Green's Theorem in the Plane. So the error minimized is around the external boundary only.

By actually computing the resultant of the errors in the normal and shearing stresses on the edges, we find that the total disturbance across the corner, which amounts to a force very nearly perpendicular to the diagonal, is compression when two terms are used and is tension when three terms are used. Thus the computed stress concentrations at the rim of the hole tend to be too large for two terms and too small for three terms. The closeness between 8.35 (two terms) and 8.316 (three terms) times the applied unit shearing stress, which are the maximum tangential normal stresses at the edge of the plain circular hole with its diameter equal to half of the panel length, shows that the effect of corner disturbance is small.

V. DISCUSSION

From observation of the theoretical results, it may be said the stress concentration factor for normal stresses is definitely reduced by reinforcing the edge of the hole in various ways. However, it is important to notice that the stress concentration factor for shearing stresses for reinforced holes always stays about the same as that for plain holes, and in certain cases becomes even greater. It indicates that no matter how much flange reinforcement around the rim of the hole is added, there must be enough net section to carry the total shear. In deciding the proper amount of reinforcing to be used, the relative magnitudes of the strength of the material in shear and in tension or compression should be considered.

The theoretical results, though giving some indication of the effects of such variables as hole-diameter, amount and ways of reinforcement, etc., must await experimental data for justification.

VITA

Chu-Kia Wang was born in Wusih, Kiangsu, China, on June 17, 1917. He graduated from St. John's University, Shanghai, China, with the degree of Bachelor of Science (with honor) in Civil Engineering in June, 1938, meanwhile winning the Gold Medal for the Best Civil Engineering Graduate. He served as Instructor in Civil Engineering in his Alma Mater for three years before he sailed for the United States to accept a fellowship at the University of Colorado in the summer of 1941, where he received the degree of Master of Science in Civil Engineering in June, 1942. He subsequently studied at the University of Illinois as Fellow in Civil Engineering from September, 1942, to May, 1943, and from October, 1944, to May, 1945. During the intervals, he was engaged in professional practice at Giffels and Vallet, Detroit, Michigan; Waddell and Hardesty, New York, New York; Curtiss-Wright Corporation, Columbus, Ohio; and Curtiss-Wright Research Laboratory, Buffalo, New York.

He was elected a Member of Phi Tau Phi, National Scholastic Honorary Society of China in April, 1940, and an Associate of Sigma Xi in May, 1942. He is an Associate Member of the Chinese Institute of Engineers and a Junior Member of the American Society of Civil Engineers.

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